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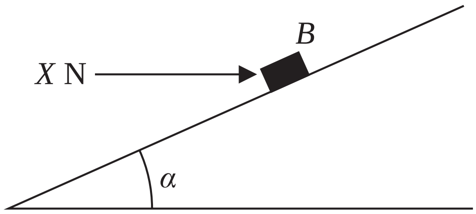


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B, **(3)**

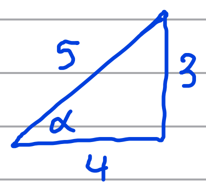
(ii) state the direction of the frictional force acting on B. **(1)**

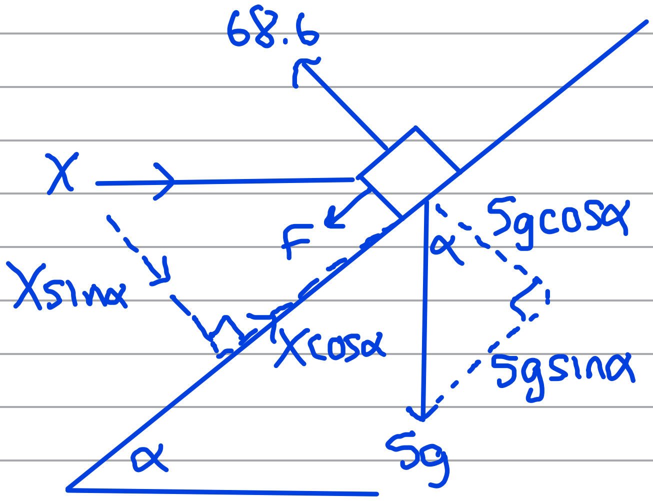
The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane. **(6)**

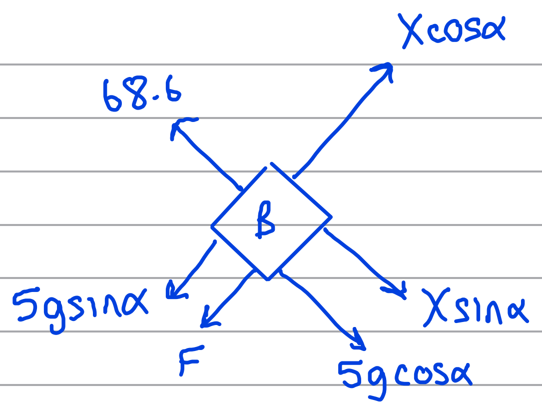
$\tan\alpha = 3/4$





$\sin\alpha = 3/5$

$\cos\alpha = 4/5$



"R( )" means resolve

a)

(i) $R(\nwarrow)$: $68.6 = X\sin\alpha + 5g\cos\alpha$ (1)

$$\Rightarrow X = \frac{68.6 - 5g\cos\alpha}{\sin\alpha} = 49\text{ N} \quad (1)$$

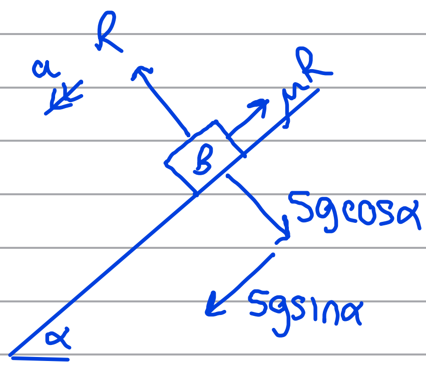
$R(\nearrow)$: $X\cos\alpha = 5g\sin\alpha + F$

$$\Rightarrow F = X\cos\alpha - 5g\sin\alpha = 9.8\text{ N} \quad (1)$$

(ii) Down the plane (1)
Friction opposes motion; without friction the box would slide up the plane, so friction must act down to counteract this.

b) $\mu = 0.5$

$F = \mu R$
$= 0.5R$ (1)



- R changes as X is removed
- friction now acts up the plane

$R(\nwarrow)$: $R = 5g\cos\alpha = 39.2$ (1) (1)

$R(\swarrow)$: $5g\sin\alpha - \mu R = 5a$ (1) (1)

$$\therefore a = \frac{5g\sin\alpha - \mu R}{5}$$

$a = 1.96\text{ ms}^{-2}$ (3sf) (1)

2 *[In this question,* **i** *and* **j** *are horizontal unit vectors.]*

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})$ N, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

a) $F_1 + F_2 = k\begin{pmatrix}3\\1\end{pmatrix}$

$\begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}3k\\k\end{pmatrix}$ 1

$4 + \lambda = 3k$ ①
$-1 + \mu = k$ ②

sub ② into ①: $4 + \lambda = 3(-1 + \mu)$ 1
$4 + \lambda = -3 + 3\mu$ 1
$\Rightarrow \lambda - 3\mu + 7 = 0$ 1

b) given $\lambda = 2$, so find μ:

$2 - 3\mu + 7 = 0$
$\mu = 3$

$\therefore$ resultant force is

$\begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$ 1

$\underline{F} = m\underline{a}$

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = 4\underline{a}$ (1)

$\underline{a} = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$

motion AB:

$\underline{s} = \underline{s}$

$\underline{u} = 0$

$\underline{v} =$

$\underline{a} = 1.5\underline{i} + 0.5\underline{j}$

$\underline{t} = 4$

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= 0 + \frac{4^2}{2}\begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$ (1)

$\underline{s} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$

u=0 as starts from rest

distance $= |\underline{s}| = \sqrt{12^2 + 4^2}$ (1) $= 4\sqrt{10}$ (1)

3.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\mathrm{m\,s^{-2}}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

(a) $R = mg$ reaction = mass × gravity

$= 5 \times 9.8$ (equal to weight, which keeps P on the plane)

$= 49\text{N}$ (1)

(b) $\overrightarrow{\text{force}} = \text{mass} \times \text{acceleration}$

$28 - F = 5 \times 1.4$ (1) total force $\rightarrow$ = 28 - F

$28 - F = 7$

$F = 21\text{ N}$ (1)

(c) $\mu = \dfrac{F}{R}$ ← friction ← normal force

$\mu = 21 \div 49$ (from part a)

$\mu = 0.43$ (1)

4

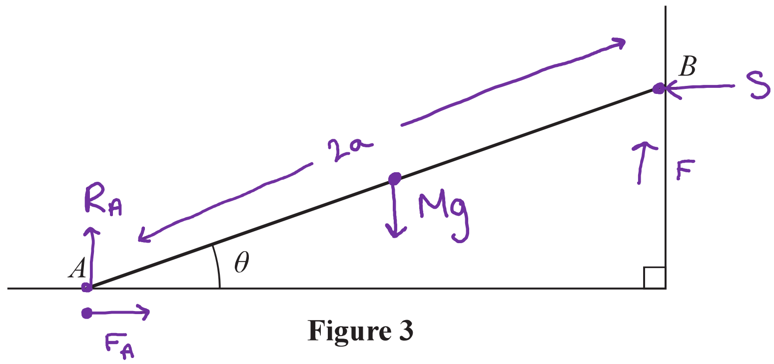


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2} Mg \cot \theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan \theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan \theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

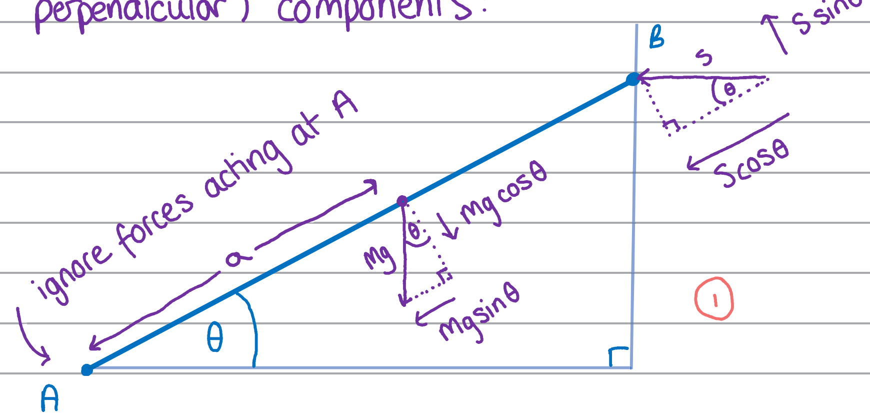
(1)

Question 4 continued

(a) Frictional force at A acts right because it must oppose the normal reaction at B, which acts left. (1)

(b) Calculate the horizontal and vertical (or parallel and perpendicular) components:

moment = force × distance from point to force



$$aMg\cos\theta = 2aS\sin\theta \quad (1)$$

$\div 2a$

$$\frac{a}{2a}Mg\cos\theta = S\sin\theta$$

$\div \sin\theta$

$$\frac{a}{2a}Mg \times \frac{\cos\theta}{\sin\theta} = S$$

$\cot = \frac{1}{\tan}$, $\tan = \frac{\sin}{\cos}$ $\Rightarrow$ $\cot = \frac{\cos}{\sin}$

$$\frac{1}{2}Mg \times \cot\theta = S \quad (1)$$

(c) Resolving vertically : $R = mg$ (1)
Resolving horizontally: $F = S$ (1)

the system is in equilibrium, so vertical and horizontal forces must be equal.

$F = \mu R \Rightarrow \mu R = S \Rightarrow \mu mg = S$ (1)

$\frac{1}{2}Mg \times \cot\theta = S$ ← from part (b)

Question 4 continued

$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$ (1) $\leftarrow$ $\tan\theta = \frac{3}{4} \Rightarrow \frac{1}{\tan\theta} = \frac{4}{3}$

$\div Mg$

$\frac{1}{2} \times \frac{4}{3} = \mu$

$\mu = \frac{2}{3}$ (1)

(d) Forces acting on A: R = normal reaction $= Mg$

$F = \mu R = \frac{2}{3}Mg$

Magnitude $= \sqrt{F^2 + R^2}$ (1)

$= \sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$ (1)

$= \sqrt{\frac{4}{9}M^2g^2 + M^2g^2}$

$= \sqrt{\frac{13}{9}M^2g^2}$

$= \frac{1}{3}Mg\sqrt{13}$ (1)

(e) New value of S would be larger because the moment of the weight about A would be larger. (1)